

# Transverse momentum distribution of vector mesons produced in ultraperipheral relativistic heavy ion collisions

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## Abstract

We study the transverse momentum distribution of vector mesons produced in ultraperipheral relativistic heavy ion collisions (UPCs). In UPCs there is no strong interaction between the nuclei and the vector mesons are produced in photon-nucleus collisions where the (quasireal) photon is emitted from the other nucleus. Exchanging the role of both ions leads to interference effects. A detailed study of the transverse momentum distribution which is determined by the transverse momentum of the emitted photon, the production process on the target and the interference effect is done. We study the total unrestricted cross section and those, where an additional electromagnetic excitation of one or both of the ions takes place in addition to the vector meson production, in the latter case small impact parameters are emphasized.

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Due to the strong electromagnetic fields surrounding the heavy ions in relativistic collisions, RHIC and LHC can be seen as a factory of quasireal photons of high energies. One of the interesting photonuclear processes studied in these “ultraperipheral collisions” (UPC) is the coherent production of vector mesons, in particular  $\rho^0$ , which has been measured recently at RHIC [1, 2]. The coherent production was identified through the transverse momentum distribution of the meson, which is enhanced for values of the transverse momentum  $v_\perp \lesssim 1/R$  where  $R$  denotes the nuclear radius. We give a careful theoretical study of the process

$$A + A \rightarrow A^{(*)} + A^{(*)} + V \quad (1)$$

with (“ $A^*$ ”) and without (“ $A$ ”) an electromagnetic (GDR) excitation of either one or both ions. This is of interest for the analysis of the RHIC experiments, as well as, for future experiments at LHC where also other vector mesons like ( $J/\Psi$  and even  $\Upsilon$ ) can be studied [3, 4, 5]. While the theory of UPC is generally in a good shape [6, 7, 8, 9], the specific question of the *transverse momentum distribution* has been paid attention to only in less rigor [7, 10, 11].

Heavy ion scattering offers a unique possibility to study an important interference effect [10]. As will be shown below, the transverse momentum distribution is very sensitive to this effect. It is the purpose of this letter to give a careful study of this transverse momentum distribution.

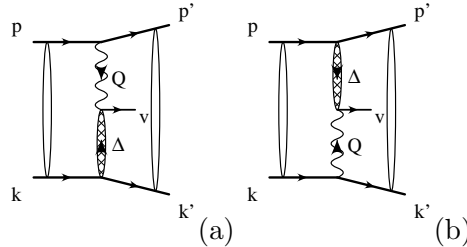


FIG. 1: A schematic Feynman diagram for the vector meson production in ultraperipheral heavy ion collisions (a). The corresponding exchange diagram is also shown (b).

The kinematics of the process given in Eq. (1) is denoted by (see Fig. 1)

$$p + k \rightarrow p' + k' + v. \quad (2)$$

Due to the additional elastic photon exchanges which are schematically denoted by the open blobs in Fig. 1 the momenta  $Q$  and  $\Delta$  are not related to the asymptotic momenta

by  $Q = p - p'$  and  $\Delta = k - k'$  as, e.g., in the  $pp$  case (without rescattering) [12]. For small transverse momenta the longitudinal components of the photon momentum and the momentum transfer from the vector meson production (“Pomeron momentum”) are given in the c.m. system by the mass  $m_V$  and the rapidity  $Y$  of the produced meson as

$$Q_0 = \beta Q_z = \frac{m_V}{2} e^Y, \quad \Delta_0 = -\beta \Delta_z = \frac{m_V}{2} e^{-Y}. \quad (3)$$

The momenta  $p$  and  $k$ , see Fig. 1(a), are given by  $p = m_A u_+$  (ion 1) and  $k = m_A u_-$  (ion 2), where  $m_A$  is the ion mass and  $u_{\pm} = \gamma(1, 0, 0, \pm\beta)$ . In the exchanged process, see Fig. 1(b), the photon is emitted from ion 2, the “Pomeron” from ion 1.

Due to the large value of the Coulomb parameter  $\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$  we can use the semiclassical approximation [2, 13, 14]. We also show how this can be derived from eikonal/Glauber theory, more details will be given in a forthcoming publication [15]. Using a simple model for the meson production process we are able to give analytical results. Implications for the current experiments at STAR/RHIC and for future experiments at the LHC [4, 10] are discussed. In the semiclassical approximation the two ions move along a straight line and the process is described by an impact parameter dependent amplitude  $a(\vec{b}, \vec{v}_{\perp}, Y)$ . In contrast to the momentum of the vector meson the momenta of the outgoing ions are not detected and the differential cross section is given by

$$\frac{d^3\sigma}{d^2v_{\perp} dY} = \frac{1}{2(2\pi)^3} \sum_{e_V} \int d^2b |a_{fi}(\vec{b}, \vec{v}_{\perp}, Y)|^2. \quad (4)$$

The integration over the impact parameter  $\vec{b}$  corresponds to an integration over the unobserved momenta  $k'$  and  $p'$  of the scattered ions. The different processes which can occur according to Eq. (1) factorise [14]:

$$a_{fi}(\vec{b}, \vec{v}_{\perp}, Y) = a_{nucl}(\vec{b}) a_1(\vec{b}) a_2(\vec{b}) a_V(\vec{b}, \vec{v}_{\perp}, Y). \quad (5)$$

The strong absorption due to the interaction of the ions for  $b < 2R$  is given by  $a_{nucl}(\vec{b}) \approx \Theta(b - 2R)$  with the nuclear radius  $R = 7\text{fm}$ .  $a_V(\vec{b}, \vec{v}_{\perp}, Y)$  describes the vector meson production. Additional electromagnetic excitation amplitudes of ion 1 and/or 2 are denoted by  $a_1(\vec{b})$  and  $a_2(\vec{b})$ . What is chosen for them depends on the additional triggering condition used in the experiment. The cross section can be written as

$$\frac{d^3\sigma}{d^2v dY} = \frac{1}{2(2\pi)^3} \int_{2R}^{\infty} d^2b f_{ij}(b) \sum_{e_V} |a_V(\vec{b}, \vec{v}, Y)|^2. \quad (6)$$

where  $f_{ij}(b)$  takes the triggering condition of the measurement into account. In the case where the process is observed without any further condition imposed on the excitation of the ion(s), we have  $f_{00}(b) = 1$ . If the vector meson production together with the electromagnetic excitation is measured one uses either  $f_{10}(b) = P_1(b) = |a_1(b)|^2$  or  $f_{01}(b) = P_2(b) = |a_2(b)|^2$  if the excitation of one of the ions,  $f_{11}(b) = P_1(b)P_2(b)$  if the mutual excitation of both ions is triggered on.

The electromagnetic excitation probabilities are given by  $P_i(b) = \frac{S}{b^2}$  with  $S \approx 5.45 \times 10^{-5} Z^3 N A^{-2/3} \text{fm}^2$  [16]. This equation is valid for  $2R < b < b_{max} = \frac{2\gamma^2 - 1}{E_{GDR}}$ , but this cutoff can be safely neglected here, since the vector meson production probability falls off at least as fast as  $1/b^2$ .

In the semiclassical treatment the amplitude of an electromagnetic process can be written as [17, 18]

$$a(\vec{b}) = \int \frac{d^4 Q}{(2\pi)^4} A_{ext}^\mu(\vec{b}, Q) J_\mu(Q), \quad (7)$$

where

$$A_{ext}^\mu(b, Q) = 2\pi Z e Q_\perp^\mu \delta(Q u_+) \frac{\gamma}{Q_0} \frac{F(Q^2)}{Q^2} \exp(-i\vec{Q}_\perp \vec{b}) \quad (8)$$

is the Liénard-Wiechert potential. A gauge transformation has been made so that the field is to a good approximation transversal.

For the elastic form factor  $F(Q^2)$  we choose  $F(Q^2) = \exp(Q^2 R_\gamma^2)$ , with  $R_\gamma \approx \sqrt{\langle r^2 \rangle / 6} \approx 2.2 \text{fm}$ . Alternatively we can set  $F(Q^2) = 1$ , that is  $R_\gamma = 0$ , as the electric field outside a spherically symmetric charge distribution is the same as that of a corresponding point charge. We find numerically the effect of a finite  $R_\gamma$  to be rather small, justifying this assumption. We still keep it for completeness in the following equations.

In order to describe the meson production we need an expression for the electromagnetic current  $J(A \rightarrow A + V)$ . This current can be found by using the vector dominance model (VDM), which relates this current to the elastic scattering amplitude  $V + A \rightarrow V + A$  as

$$J^\mu(Q) = e_V^\mu C_V f_{el}(\Delta_\perp, Y) \quad (9)$$

with  $\Delta_\perp = v_\perp - Q_\perp$ .  $e_V$  is the polarisation of the outgoing vector meson, which by assuming  $s$ -channel helicity conservation (SCHC) is identical to the one of the incoming photon, see [19] for details.  $C_V$  describes the vector meson content of the photon, see e.g. [5].

In the following we choose the elastic vector meson scattering amplitude as

$$f_{el}(\Delta_\perp, Y) = f_0(Y) \exp(-\Delta_\perp^2 R_V^2) \delta(\Delta u_-) (v u_-) \quad (10)$$

with  $R_V \approx 2.2\text{fm}$  to reproduce the slope of the angular distribution. This form agrees also with the one proposed in [20]. It has been mainly chosen for ease of calculations, whereas the formalism given can be extended to more realistic forms, e.g., based on a full eikonal description. A simple extension is possible by using a sum of Gaussians for  $f_{el}$ .

The imaginary part of  $f_0$  can be related to the total cross section for vector meson scattering on nuclei through the optical theorem. For the energies of RHIC and LHC the real part is small (of the order of 10%). While for  $Y \neq 0$  (and asymmetric collisions) there is a sensitivity to the phase of  $f_0$ , for rapidity  $Y = 0$  we only need the absolute value of  $|C_V f_0|$ . This we choose in order to reproduce the cross section given in [3, 21] as  $d\sigma/dY_\rho(\text{RHIC}) = 70\text{mb}$  and  $d\sigma/dY_{J/\Psi}(\text{LHC}) = 0.75\text{mb}$ .

Using these expressions we get for the amplitude:

$$a_V(\vec{b}, \vec{v}_\perp, Y) = \frac{ZeC_V f_0}{(2\pi)^3} \exp(-Q_l^2 R_\gamma^2) \int d^2 Q_\perp (\vec{Q}_\perp \vec{e}_V) \frac{\exp(-Q_\perp^2 R_\gamma^2)}{Q_\perp^2 + Q_l^2} \exp(-i\vec{Q}_\perp \vec{b}) \exp(-R_V^2 (\vec{v}_\perp - \vec{Q}_\perp)^2) \quad (11)$$

with  $Q_l^2 = Q_z^2 - Q_0^2 = (\frac{Q_0}{\beta_\gamma})^2$ . Let us first make a short qualitative discussion: if we would neglect  $\vec{Q}_\perp$  in  $\exp(-R_V^2 (\vec{v}_\perp - \vec{Q}_\perp)^2)$ , the dependence of  $|a_V|^2$  and  $d^3\sigma/d^2v_\perp dY$  on  $v_\perp$  would be of the form  $\exp(-2R_V^2 v_\perp^2)$ , which is due to  $J$  alone. This coincides with the result for an incident photon of zero transverse momentum. The effect of the finite  $Q_\perp$  distribution of the photon is to broaden this distribution. As the width of the  $Q_\perp$  distribution depends on  $b$  via  $\exp(-i\vec{Q}_\perp \vec{b})$ , the effect of this broadening will depend on  $b$ . This effect is largest for small  $b$ , as the perpendicular momentum distribution of the photon is of the order  $1/b$ .

An analytic approximation for  $a_V$  can be found in the region of small  $b$ , that is if  $b < 1/Q_l$  and  $Q_l$  can be neglected in the photon propagator in Eq. (11), which corresponds to the sudden limit. One gets (see [15] for details):

$$a_V(\vec{b}, \vec{v}_\perp, Y) \approx \frac{ZeC_V f_0 i}{(2\pi)^2} \frac{(\vec{b} + 2i\vec{v}_\perp R_V^2) \vec{e}_V}{(\vec{b} + 2i\vec{v}_\perp R_V^2)^2} \exp(-Q_l^2 R_\gamma^2) \exp(-v_\perp^2 R_V^2) \left[ \exp\left(\frac{-(\vec{b} + 2i\vec{v}_\perp R_V^2)^2}{4(R_V^2 + R_\gamma^2)}\right) - 1 \right]. \quad (12)$$

The same final state can be obtained by exchanging the roles of both ions, see Figs. 1(a) and (b). The corresponding amplitude  $a_V^X(\vec{b}, \vec{v}_\perp, Y)$  where the photon is emitted from ion 2

and the ‘‘Pomeron’’ from ion 1 is given by

$$a_V^X(\vec{b}, \vec{v}_\perp, Y) = \int \frac{d^4 Q}{(2\pi)^2} A_{ext}^\mu(0, Q) J_\mu^X(Q), \quad (13)$$

where the impact parameter for  $A_{ext}$  is now  $\vec{b} = 0$  and  $u_+$  is replaced by  $u_-$ . The electromagnetic current  $J^X$  is now for vector meson production on an ion at position  $\vec{b}$ . One finds

$$\begin{aligned} J_\mu^X(Q) &= J_\mu(Q) \exp(-i\vec{Q}_\perp \vec{b}) \\ &= J_\mu(Q) \exp(-i\vec{v}_\perp \vec{b} + i\vec{\Delta}_\perp \vec{b}). \end{aligned} \quad (14)$$

We find that the exchange amplitude is of the form:

$$a_V^X(\vec{b}, \vec{v}_\perp, Y) = a_V(-\vec{b}, \vec{v}_\perp, -Y) \exp(-i\vec{v}_\perp \vec{b}). \quad (15)$$

This has a simple interpretation: In order to exchange the role of the two ions,  $Y$  is replaced by  $-Y$ , the direction of  $\vec{b}$  needs to be reversed and in addition the origin needs to be shifted by  $\vec{b}$ , leading to the extra phase  $\exp(-i\vec{v}_\perp \vec{b})$ . This relation was also used in [10, 22]. With  $a_V$  from Eq. (11) we finally get

$$a_V^{tot}(\vec{b}, \vec{v}_\perp, Y) = a_V(\vec{b}, \vec{v}_\perp, Y) + e^{-i\vec{v}_\perp \vec{b}} a_V(-\vec{b}, \vec{v}_\perp, -Y). \quad (16)$$

The analytic expression in Eq. 12 allows us to discuss some properties of the transverse momentum distribution of the process: In the limit  $v_\perp R_V^2 \ll b$  one has  $a_V \sim \vec{b} \vec{e}_V$  and  $a_V^X = -a_V$ , i.e., the amplitudes have a relative sign of  $-1$ , leading to destructive interference at small  $b$ . In the other limit  $v_\perp R_V^2 \gg b$  one has  $a_V^X = a_V$ , i.e., the same relative sign, but  $a_V$  and  $a_V^X$  are smaller than in the first case due to the last exponential in Eq. (12). The transverse momentum distribution is therefore more complex than treated in [10, 22].

We can also derive the results starting from the eikonal or Glauber approach to multi-photon processes in UPC collisions, see [14]. In this case the scattering amplitude is given by

$$f_{fi, \text{Glauber}}(\vec{K}) = \frac{i\pi}{k} \int d^2 b \exp(i\vec{K} \vec{b}) \langle f | \exp(i\chi(\vec{b})) | i \rangle, \quad (17)$$

where  $\vec{K} = \vec{k}' - \vec{k}$  is the total momentum transfer to the ‘‘target’’ nucleus. The eikonal  $\chi(b)$  takes care of all the different elastic and inelastic processes. In our case we have

$$\chi(\vec{b}) = \chi_{nuc}(b) + \chi_C(b) + \chi_1(b) + \chi_2(b) + \chi_V(\vec{b}). \quad (18)$$

The term  $\chi_{nuc}(b)$  describes the effect coming from the nuclear interaction between the two ions. It can be approximated by  $\exp(i\chi_{nuc}(b)) \approx \Theta(b-2R)$ . The term  $\chi_C \approx \exp(2i\eta \log(kb))$  describes the elastic Coulomb scattering. The last three terms describe the additional electromagnetic interactions: the possible excitation of the first and second nucleus and the vector meson production. The eikonal phase for the vector meson production process  $\chi_V(b)$  can usually be treated in lowest order by expanding the exponential. The second order term would describe double  $\rho$  production, which is still sizeable, see [11]. Bracketing with the initial and final states we get

$$\begin{aligned} \langle f | \exp(i\chi(\vec{b})) | i \rangle &\approx i \exp(i\chi_{nuc}(b)) \exp(+i\chi_C(b)) \\ \langle f_1 | \exp(i\chi_1(b)) | i_1 \rangle &\langle f_2 | \exp(i\chi_2(b)) | i_2 \rangle \\ \langle V, i_2 | \chi_V(\vec{b}) | i_2 \rangle & \end{aligned} \quad (19)$$

with  $|i\rangle = |i_1, i_2\rangle$  the initial (ground) states of the two ions and  $|f\rangle = |f_1, f_2\rangle |V\rangle$  the final states of the ions and the meson. In order for this process to factorise, we made the reasonable assumption, that the vector meson production on the excited nucleus is the same as the one on the nucleus in the ground state:

$$\langle V, i_2 | \chi_V(b) | i_2 \rangle \approx \langle V, f_2 | \chi_V(b) | f_2 \rangle . \quad (20)$$

There is a correspondence of these terms to the different semiclassical amplitudes  $a_{fi}(b)$  in Eq. (5), which was also explored in [23]. The  $\chi$  is given by

$$\chi(\vec{b}) = -1 \int \frac{d^4Q}{(2\pi)^4} A_{\text{eik}}^\mu(\vec{b}, z, Q) \hat{J}_\mu(Q) \quad (21)$$

with  $A_{\text{eik}}^\mu$  as given in Eq. (8) with  $\delta(Qu_+)$  replaced with  $\delta(\gamma(Q_0 - Q_z))$ , which corresponds to the expression of the semiclassical amplitude  $a_V$  in Eq. (7) in the sudden limit.

The major difference between the two approaches is the presence of the Coulomb eikonal  $\chi_C(b)$ . For  $\eta \gg 1$ ,  $\chi_C(b)$  is a rapidly varying function and one can evaluate the Glauber expression by means of the well known saddle point approximation. One obtains the relation between the classical impact parameter and the momentum transfer:  $b = 2\eta/K$ . As the momentum transfer to the ion is not measured in our case, one calculates “inclusive” cross sections by integrating over  $K$ . This gives the same result as in the semiclassical case of Eq. (5) in the sudden limit ( $Q_t = 0$ ), as the Coulomb phase is purely imaginary and the

integration over  $K$  leads, (see Eq. (10)-(12) of [23]) to a  $\delta$ -function for the two impact parameters in  $|f_{fi,\text{Glauber}}|^2$ .

We use both the exact expression Eqs. (6) and (11), as well as, the approximate analytical result Eq. (12) to calculate results for the case studied in [10]. They are shown in Fig. 2. The analytic result is too large in the untagged case, but its shape agrees quite nicely with the full calculation. The effect of tagging for small  $b$  is a shift of the maximum of the curve to larger values of  $v_\perp$  and a more pronounced interference structure. In Fig. 3 we also show the similar results for  $J/\Psi$  production at the LHC.

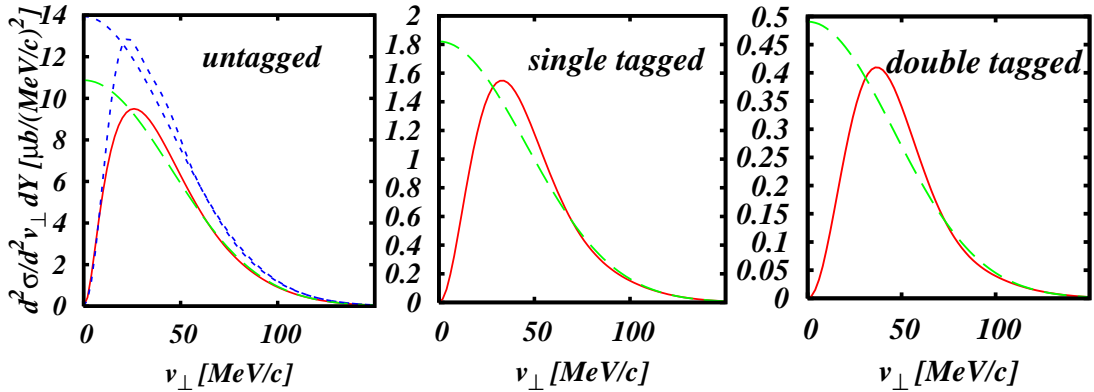


FIG. 2: The differential cross section  $d\sigma/d^2v_\perp dY$  is shown for  $\rho^0$  production at  $Y = 0$  at RHIC (Au-Au collisions with  $\gamma = 108$ ). The solid line is the result including the interference, the dashed line the result from an incoherent adding of the two processes. The results for the three different tagging cases are given in (a), (b) and (c). The approximate result is shown as dotted line only in the first (untagged) case. In the other two case it cannot be distinguished from the full calculation.

Let us summarize our findings: We have put the transverse momentum distribution on a firm theoretical basis starting our derivation from the semiclassical approximation or alternatively from Glauber theory. The meson transverse momentum distribution was derived as a function of  $b$  and an analytic expression was given. The interference phenomenon was derived within this model. As the main outcome we find in this letter that for a good understanding of the interference phenomenon a careful study of the transverse momentum distribution is essential. Whereas formally the results look similar to the one given in [2, 10], differences appear both in the transverse momentum distribution as a function of  $b$  and in the form of the interference. This leads to a more complex result in the intermediate  $v_\perp$  region. This will also be true in the case, where one is moving away from  $Y = 0$ . Our findings



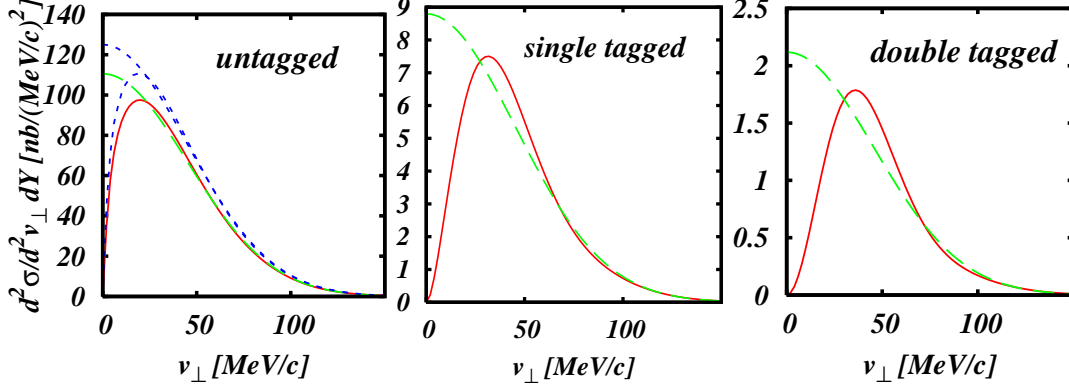


FIG. 3: The differential cross section  $d\sigma/d^2v_\perp dY$  is shown for  $J/\Psi$  production at  $Y = 0$  at LHC (Pb-Pb ions with  $\gamma = 3000$ ). The lines are the same as in Fig. 2.

are important in analyzing the experimental data of STAR and also PHENIX. Results have also been given for future LHC measurements, which would be even more interesting for  $J/\Psi$  or even  $\Upsilon$  production.

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Figure 2:

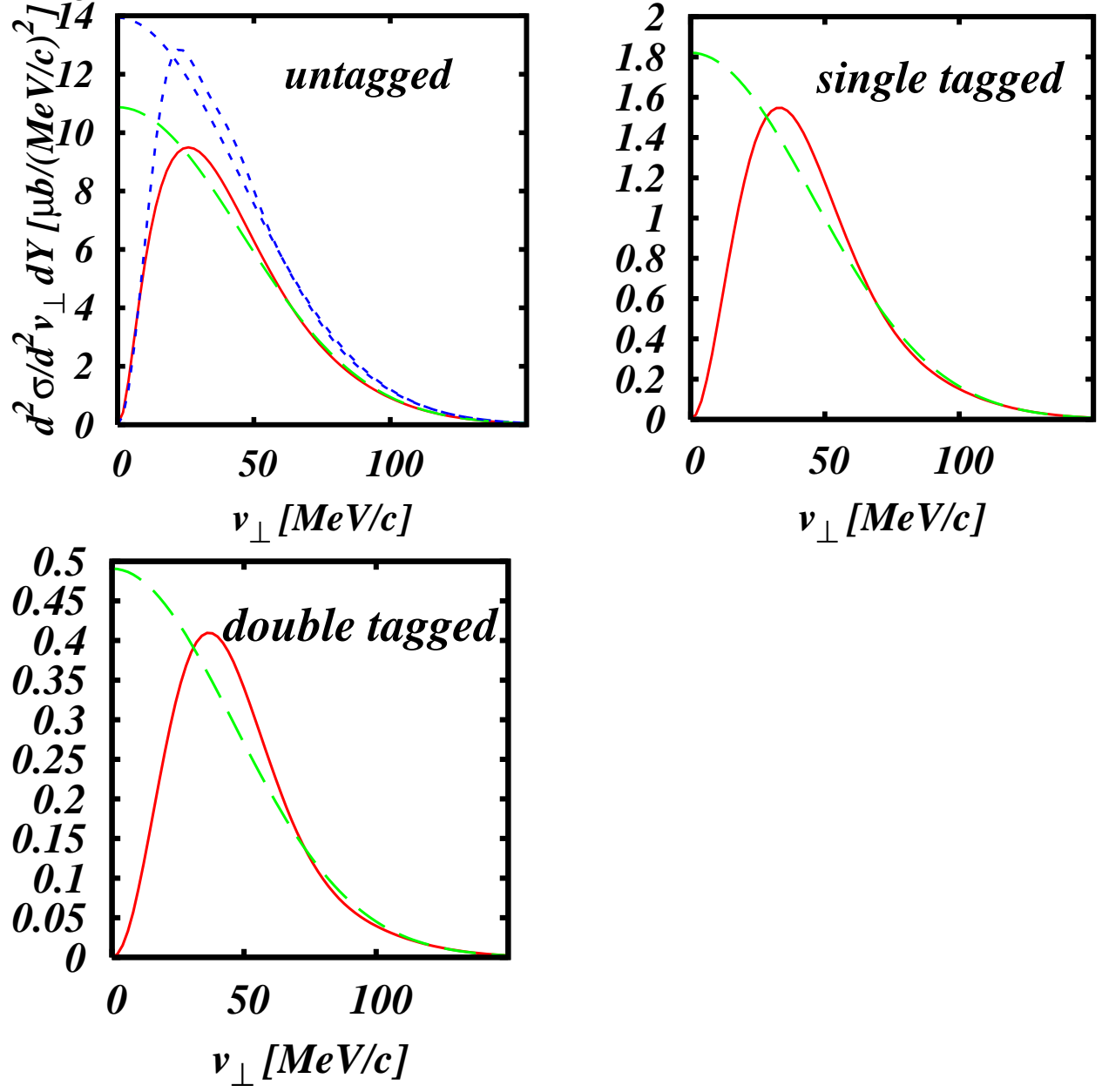


Figure 3:

